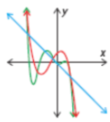
Unit 1 Review - Intro to Polynomial Functions

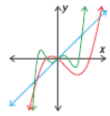
• Polynomial functions of the same degree have similar characteristics.
• The degree and the leading coefficient in the equation of a polynomial function indicate the end behaviours of the graph.
• The degree of a polynomial function provides information about the shape, turning points, and zeros of the graph.

End Behaviours

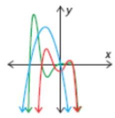
- An odd-degree polynomial function has opposite end behaviours.
  - If the leading coefficient is negative, then the function extends from the second quadrant to the fourth quadrant; that is, as $x \to -\infty$, $y \to \infty$ and as $x \to \infty$, $y \to -\infty$.



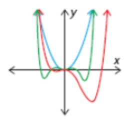
  - If the leading coefficient is positive, then the function extends from the third quadrant to the first quadrant; that is, as $x \to -\infty$, $y \to -\infty$ and as $x \to \infty$, $y \to \infty$.



- An even-degree polynomial function has the same end behaviours.
  - If the leading coefficient is negative, then the function extends from the third quadrant to the fourth quadrant; that is, as $x \to \pm\infty$, $y \to -\infty$.



  - If the leading coefficient is positive, then the function extends from the second quadrant to the first quadrant; that is, as $x \to \pm\infty$, $y \to \infty$.



Turning Points
• A polynomial function of degree *n* has at most n-1 turning points.

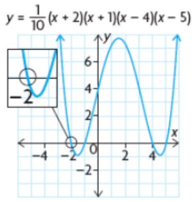
Number of Zeros
• A polynomial function of degree *n* may have up to *n* distinct zeros.
• A polynomial function of odd degree must have at least one zero.
• A polynomial function of even degree may have no zeros.

Symmetry
• Some polynomial functions are symmetrical in the *y*-axis. These are even functions, where f(-x) = f(x).
• Some polynomial functions have rotational symmetry about the origin. These are odd functions, where f(-x) = -f(x)
• Most polynomial functions have no symmetrical properties. These are functions that are neither even nor odd, with no relationship between *f*(-x) and *f*(*x*).

• The zeros of the polynomial function, y = f(x) are the same as the roots of the related polynomial equation, f(x) = 0

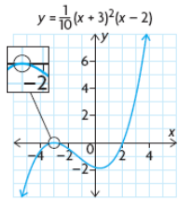
- To determine the equation of a polynomial function in factored form, follow these steps:
  - Substitute the zeros $(x_1, x_2, \ldots, x_n)$ into the general equation of the appropriate family of polynomial functions of the form $y = a(x - x_1)(x - x_2)\ldots(x - x_n)$.
  - Substitute the coordinates of an additional point for x and y, and solve for a to determine the equation.
- If any of the factors of a polynomial function are linear, then the corresponding x-intercept is a point where the curve passes through the x-axis. The graph has a linear shape near this x-intercept.

$y = \frac{1}{10}(x + 2)(x + 1)(x - 4)(x - 5)$



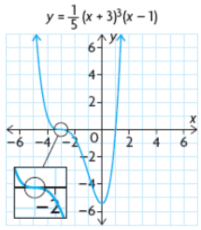
- If any of the factors of a polynomial function are squared, then the corresponding x-intercepts are turning points of the curve and the x-axis is tangent to the curve at these points. The graph has a parabolic shape near these x-intercepts.

$y = \frac{1}{10}(x + 3)^2(x - 2)$



- If any of the factors of a polynomial function are cubed, then the corresponding x-intercepts are points where the x-axis is tangent to the curve and also passes through the x-axis. The graph has a cubic shape near these x-intercepts.

$y = \frac{1}{5}(x + 3)^3(x - 1)$



• Polynomials can be divided in much the same way that numbers are divided.

• A polynomial can be divided by a polynomial of the same degree or less.
• Synthetic division is a shorter form of polynomial division. It can only be used when the divisor is linear, that is (x - k) or (ax - k).
• When using polynomial or synthetic division,
• terms should be arranged in descending order of degree, in both the divisor and the dividend, to make the division easier to perform
• zero must be used as the coefficient of any missing powers of the variable in both the divisor and the dividend
• If the remainder of polynomial or synthetic division is zero, both the divisor and the quotient are factors of the dividend.

- The remainder theorem: When a polynomial, $f(x)$, is divided by $x - a$, the remainder is equal to $f(a)$.
- The factor theorem: $x - a$ is a factor of $f(x)$, if and only if $f(a) = 0$.

- To factor a polynomial, $f(x)$, of degree 3 or greater,
  - use the Factor Theorem to determine a factor of $f(x)$
  - divide $f(x)$ by $x - a$
  - factor the quotient, if possible
- If a polynomial, $f(x)$, has a degree greater than 3, it may be necessary to use the factor theorem more than once.
- Not all polynomial functions are factorable.

- An expression that contains two perfect cubes that are added together is called a sum of cubes and can be factored as follows:

  $A^3 + B^3 = (A + B)(A^2 - AB + B^2)$

- An expression that contains perfect cubes where one is subtracted from the other is called a difference of cubes and can be factored as follows:

  $A^3 - B^3 = (A - B)(A^2 + AB + B^2)$

- The solutions to a polynomial equation $f(x) = 0$ are the zeros of the corresponding polynomial function, $y = f(x)$.

• Polynomial equations can be solved using a variety of strategies:
• algebraically using a factoring strategy
• graphically using a table of values, transformations, or a graphing calculator
• Only some polynomial equations can be solved by factoring, since not all polynomials are factorable. In these cases, graphing technology must be used.
• When solving problems using polynomial models, it may be necessary to ignore the solutions that are outside the domain defined by the conditions of the problem.

To solve a polynomial inequality algebraically, you must first determine the roots of the corresponding polynomial equation. Then you must consider the sign of the polynomial in each of the intervals created by these roots. The solution set is determined by the interval(s) that satisfy the given inequality.

- Some polynomial inequalities can be solved algebraically by
  - using inverse operations to move all terms to one side of the inequality
  - factoring the polynomial to determine the zeros of the corresponding polynomial equation
  - using a number line, a graph, or a factor table to determine the intervals on which the polynomial is positive or negative
- All polynomial inequalities can be solved using graphing technology by
  - graphing each side of the inequality as a separate function
  - determining the intersection point(s) of the functions
  - examining the graph to determine the intervals where one function is above or below the other, as required
    or
  - creating an equivalent inequality with zero on one side
  - identifying the intervals created by the zeros of the graph of the new function
  - finding where the graph lies above the x-axis (where $f(x) > 0$) or below (where $f(x) < 0$), as required